

Pre Calculus

Date:

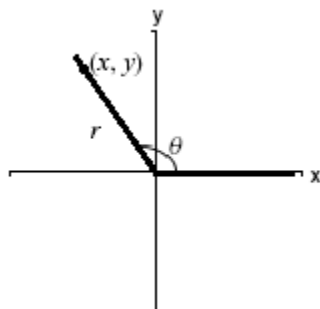
Items Needed: .Book, triangle graph handout, overhead

Objective: The students will be able to find the trig functions of any angle using triangles other than a unit circle and using reference angles to determine equivalent angles.

PA Common Core: cc.2.2.hs.c.7, cc.2.2.hs.c.7

Lesson:

- Remind students from the last section about how we could calculate trig functions using x and y values of a unit circle. $\cos \theta = \frac{x}{1}$, $\sin \theta = \frac{y}{1}$, $\tan \theta = \frac{y}{x}$ and just remember that the other functions are the reciprocals of the basic functions. Since the radius of a unit circle is 1 the hypotenuse will always be 1.
- We will not always be able to deal with a unit circle. This means that the radius will not always be 1.



- Based upon this drawing how would we go about finding the trig functions. We need to just calculate r and then we can calculate the trig functions just the same as we did before.
- Do example 1 in book. Extra example if needed.

Example 1. Let $(4, -3)$ be on the terminal side of θ . Find the value of the sine, cosine, and tangent of θ

$$r = \sqrt{4^2 + (-3)^2} = 5. \text{ So, } \sin \theta = -3/5, \cos \theta = 4/5, \text{ and } \tan \theta = -3/4.$$

- Discuss the signs of the trigonometric functions in the four quadrants. Because both x and y are positive in the first quadrant, all six functions are positive in the first quadrant. Because only y is positive in the second quadrant, only sine and cosecant are positive in the second quadrant. Since both x and y are negative in the third quadrant, only tangent and cotangent are positive in the third quadrant. Since only x is positive in the fourth quadrant, only cosine and secant are positive in the fourth quadrant.
- Do example 2 in book. Extra example if needed.

Example 2. Given $\cos \theta = 3/5$ and $\tan \theta < 0$, find $\sin \theta$ and $\cot \theta$. θ must be in the fourth quadrant. Using $x = 3$, $y = -4$, and $r = 5$, we have $\sin \theta = -4/5$ and $\cot \theta = -3/4$.

- What are the trig values of the quadrant angles?
- Remind the students about the idea of a terminal and coterminal angles and how they have the same values for each of the trig functions.
- We also have something on the same idea called a reference angle which is defined as: Let θ be an angle in standard position. Its reference angle is the acute angle θ' formed by the terminal side of θ and the horizontal axis (x-axis).
- Do an outline similar to figure 4.37 on p. 286 and then do some examples.
 1. Determine what quadrant the angle is in (radian or degrees).
 2. Determine whether the angle is closer to π than to 2π .
 3. Subtract accordingly.

Example 3. Find the reference angle for the following angles.

a) $\theta = 125^\circ$
 $\theta' = 180^\circ - 125^\circ = 55^\circ$

b) $\theta = 5$
 $\theta' = 2\pi - 5 \approx 1.2832$

- Show how the angle and the reference yield equivalent answers with the exception of the sign.
- Determine what quadrant θ is in from the previous examples and remember the signs of the different trig functions in that particular quadrant.
- This means that to evaluate trigonometric functions of any angle, we need only find the value of that function for the reference angle and attach the proper sign, according to the quadrant in which θ lies.

Example 4. Evaluate the following trigonometric functions.

a) $\sin 210^\circ$. Since 210° is in quadrant III and the reference angle is 30° , $\sin 210^\circ = -\sin 30^\circ = 1/2$.

b) $\cos (-5\pi/4)$. Since $-5\pi/4$ is in the second quadrant and the reference angle is $\pi/4$, $\cos\left(-\frac{5\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

c) $\tan 5\pi/3$. Since $5\pi/3$ is in the fourth quadrant and the reference angle is $\pi/3$, $\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$.

- **Teach study tip to help remember the sine and cosine exact values.**
- Do example 6 p. 288 in the book using a right triangle and then show by using trig identities.

Assignment: Have students do 7-10, 12-21 (every 3), 23-26, p. 289.

Have students do 27, 30, 33, 39-57 (every 3), 65, 66-84 (every 3), p. 290.

Have students do 93-102 (every 3), 104, 107, 123, 125 p. 290

Evaluation: (Could be from any one/several of the following)

Responses from classroom questions
Results of classroom sample problems
Homework responses
Check answer with Calculator
End of the section exam

Enrichment: