Pre Calculus

Date:

Items Needed: .Book, blank unit circle

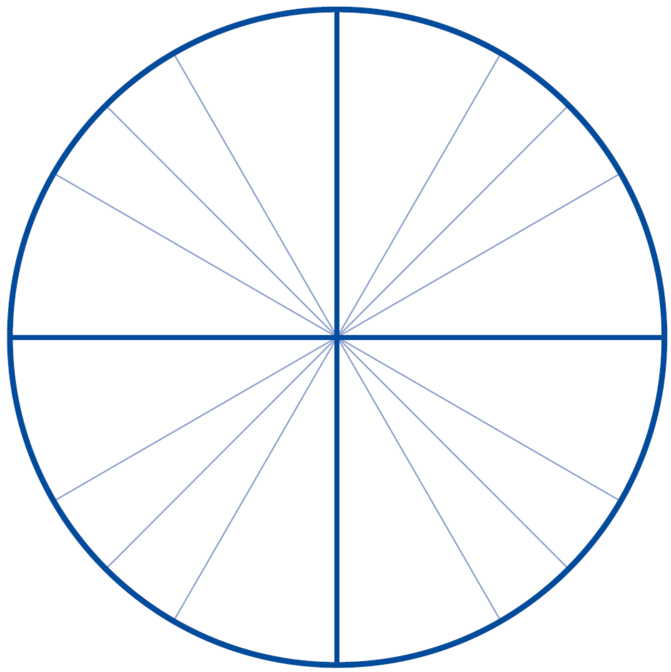
Objective: The students will be able to describe an angle and to convert between degree and radian measures.

PA Common Core: cc.2.2.hs.c.7, cc.2.3.hs.a.9

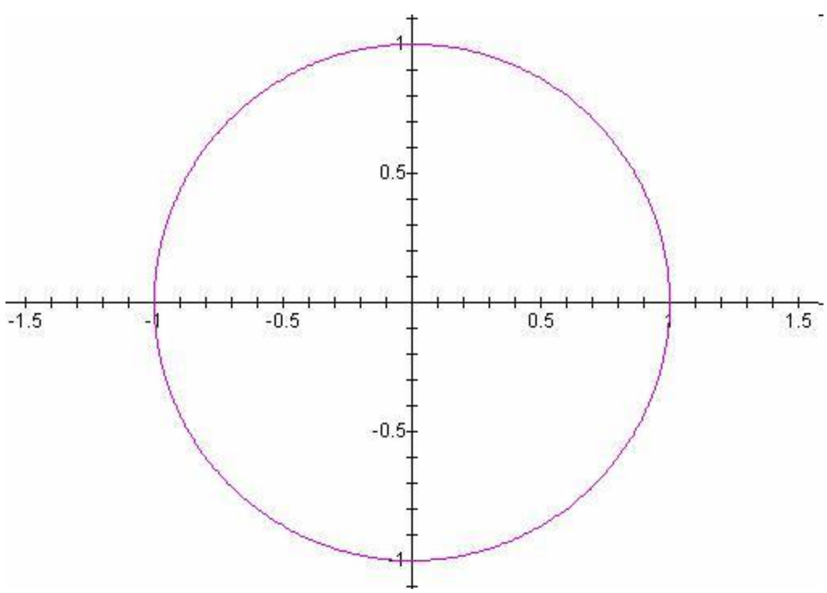
Lesson:

- An **angle** is two rays with the same initial point.
- The **measure of an angle** is the amount of rotation required to rotate one side, called the **initial side**, to the other side, called the **terminal side**.
- The shared initial point of the two rays is called the **vertex** of the angle.
- An angle is in **standard position** if its vertex is at the origin of the rectangular coordinate system and its initial side lies along the positive *x*-axis.
- If the rotation of an angle is in the counterclockwise direction, then the angle is said to be **positive.** If the rotation is clockwise, then the angle is **negative.**
- Two angles in standard position that have the same terminal side are said to be **coterminal.**

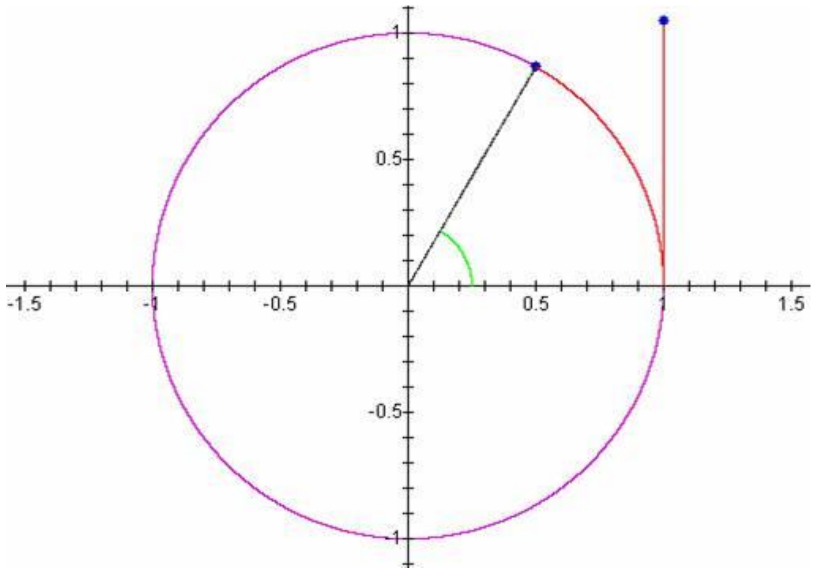
- The measure of an angle is determined by the amount of rotation from the initial side to the terminal side. One way to measure angles is in degrees. Put a blank unit circle on the board label each angle accordingly.



- First, we need to understand the unit circle. As the name implies, it is a circle where the radius is 1 (one unit). The unit circle equation is $x^2 + y^2 = 1$ and the graph looks like this:



- Now, if we start at the point (1, 0) and walk a distance t around the circle, we will arrive at a point (x, y) represented by the blue point on the circle in the figure below. The distance traveled, t, is shown in red. At the right of the circle is a red line segment that ends in a blue point that is exactly the same length as the red arc ending in the blue point.



- **One radian** is the measure of central angle θ that intercepts an arc *s* equal in length to the radius *r* of the circle.
- Look at the definition on p. 255.

- If the angle above was approximately 57.30 degrees, then the radius length would equal the arc length and that would be considered to be 1 radian. (Look at the conversion later in the section.)

- How do you find the circumference of a circle? $circ = \pi d = 2\pi r$
- This means that there are approximately 6.28 radius lengths in a full circle.

- Using the blank unit circle that we used a little while ago, match the corresponding radian measures with the degree measures that we already listed.
- Think in terms of 2π and split it into quarters and then thirds.

- All these values can then be labeled in a particular quadrant.
- The values on the axes are not considered to be in a quadrant.

- Also state that angles with measure between 0 and $\pi / 2$ are called **acute.** Angles with measure between $\pi / 2$ and π are called **obtuse.** Furthermore, two positive angles are said to be **complementary** if the sum of their measures is $\pi / 2$, and **supplementary** if the sum of their measures is π.

Example 1. Find the complement and supplement of a $\pi/5$ angle.

Complement: $\pi/2 - \pi/5 = 3\pi/10$

Supplement: $\pi - \pi/5 = 4\pi/5$

- Discuss how to find coterminal angles using example 1 p. 256.
- The measure of an angle is determined by the amount of rotation from the initial side to the terminal side. Another way to measure angles is in degrees.

- A measure of one degree is equivalent to a rotation of 1/360 of a complete revolution about the vertex.
- $360° = 2\pi$ rad. and $180° = \pi$ rad.
- Talk about how to convert from one measure to another.
- **When no units of angle measure are specified, radian measure is implied.**
- Do examples.

Example 2. Convert the following degree measures to radian measure.

a) $120° = 120\left(\frac{\pi}{180}\right) = \frac{2\pi}{3}$

b) $-315° = -315\left(\frac{\pi}{180}\right) = \frac{7\pi}{4}$

Example 3. Convert the following radian measures to degrees.

a) $\frac{5\pi}{6} = \frac{5\pi}{6}\left(\frac{180°}{\pi}\right) = 150°$

b) $7 = 7\left(\frac{180°}{\pi}\right) = \frac{1260}{\pi} \approx 401.07°$

- Define arc length as being $s = r\theta$ where θ is measured in radians.
- Make sure you change θ into radians first before you do the calculations.
- Do this example – A circle has a radius of 6 inches. Find the length of the arc intercepted by a central angle of 210 degrees.

- Define Linear speed $= \frac{s}{t}$ where t is time.
- Define Angular speed as $angularspeed = \frac{\theta}{t}$
- State that because $s = \theta r$, $\frac{s}{t} = r\frac{\theta}{t}$, where t is time (divide both sides by t)

  $linearspeed = radius \cdot angularspeed$ then $\frac{linearspeed}{radius} = angularspeed$
- Do example 6 & 7.
- Do this example - An automobile is traveling at 45 mph. If its tires have a **radius** of 15 inches, at what rate, in rpms, are the tires spinning? (Change 45 to feet/min then find the circumference of the tire and divide the two values) If you want to find the angular speed just take the number of revolutions time 2 pie and then divide by the unit of time.

Assignment: .Have students do 2-10, 12-27 (every 3), 28, 30, 33-54 (every 3), p. 261.
Have students do 57-66 (3), 68, 70, 72, 78, 79, 82,
Have students do 84-92 (even), 94, 98, 102, 108- 115, p. 262

Evaluation: (Could be from any one/several of the following)

Responses from classroom questions
Results of classroom sample problems
Homework responses
Check answer with Calculator

End of the section exam

Enrichment: